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# A Generalized Adaptive Dynamics Framework can Describe the Evolutionary Ultimatum Game

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Adaptive dynamics describes the evolution of games where the strategies are continuous functions of some parameters. The standard adaptive dynamics framework assumes that the population is homogeneous at any one time. Differential equations point to the direction of the mutant that has maximum payoff against the resident population. The population then moves towards this mutant. The standard adaptive dynamics formulation cannot deal with games in which the payoff is not differentiable. Here we present a generalized framework which can. We assume that the population is not homogeneous but distributed around an average strategy. This approach can describe the long-term dynamics of the Ultimatum Game and also explain the evolution of fairness in a one-parameter Ultimatum Game.



## 1. Introduction

In the Ultimatum Game (Güth *et al.*, 1982) two players have to split a sum of money. The first player, called the proposer, decides what proportion he will offer to the other player. The second player, the responder, simply agrees to the split or does not. If he does, they split the sum as agreed; if he does not, then neither player receives any payoff. Game theory predicts that rational players will offer and accept next to nothing. A rational responder should prefer the smallest positive share to getting nothing at all. Hence, a rational proposer should offer the smallest positive share. This is the so-called subgame perfect equilibrium solution to the Ultimatum Game (Rubinstein, 1982; Binmore, 1998).

In experimental studies of human players, however, the average offers and aspiration levels are in the region of 40–50% (Güth & Tietze, 1990; Thaler, 1988; Roth *et al.*, 1991; Fehr & Gächter, 1999; Fehr & Schmidt, 1999; Gintis, 2000). The question arises as to why humans disregard the rational solution in favor of some notion of fairness.

A common explanation is that players have preferences which depend on factors other than their own payoff, and that responders are ready to punish proposers offering only a small share by rejecting the deal. But how do these preferences arise? The most frequent explanation is that the players do not realize that they interact only once. Humans are used to repeated interactions. Repeating the Ultimatum Game is like haggling over a price, and fair splits are more likely (Roth, 1995; Rubinstein, 1982; Bolton, 1991). Another explanation is based on the view that allowing a co-player to get a large share is giving a relative advantage to a direct rival. This argument holds only for very small groups; however, responders should only reject offers that are less than $1/N$-th of the total sum, where $N$ is the number of individuals in the group (Huck & Öchssler, 1999). A third explanation is based on the idea that a substantial proportion of

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humans maximize a subjective utility function different from the payoff (Kirchsteiger, 1994; Bethwaite & Tompkinson, 1996; Fehr & Schmidt, 1999).

In previous papers (Page *et al*., 2000; Nowak *et al*., 2000), we have shown that evolutionary game theory also predicts the emergence of the rational solution for the basic Ultimatum Game. The payoff of the game is interpreted as fitness, that is reproductive success. Successful strategies spread either by biological or cultural reproduction. The intuitive explanation of the observed evolutionary dynamics is as follows: since it is costly to reject offers, responders should be willing to accept low offers. In turn, however, since it is costly to make unnecessarily high offers, proposers should tend to offer as little as possible. Therefore, natural (or cultural) selection will drive offer and acceptance levels to very low values (precisely to the point where the average offer is about the inverse of the population size). Page *et al*. (2000) show, however, that spatial effects promote the evolution of fairness, while Nowak *et al*. (2000) show that reputation (that is information about previous interactions) can lead to fair solutions.

The standard framework to describe evolutionary game dynamics is given by the replicator equation (Taylor & Jonker, 1978; Hofbauer *et al*., 1979; Hofbauer & Sigmund, 1998). In this approach, selection operates on a fixed number of strategies. Asymptotically stable, fixed points of the replicator equation correspond to evolutionarily stable strategies (Maynard Smith, 1982) or Nash equilibria (Nash, 1996). Mutation or continuous emergence of new strategies is normally not considered.

Another approach for evolutionary games is given by the adaptive dynamics framework (Nowak & Sigmund, 1990; Metz *et al*., 1996; Geritz *et al*., 1997, 1998; Dieckmann, 1997; Hofbauer & Sigmund, 1998). Here it is assumed that all players of the population use the same strategy at any one time. Mutation generates variant strategies that are close to the resident strategy. If a mutant cannot invade, it is discarded. If, however, the mutant can invade and takeover, the whole population will adopt the mutant strategy. Adaptive dynamics can be formulated as a differential equation in the space of all strategies. Evolutionarily stable strategies correspond to the equilibria of the adaptive dynamics; they can be stable or unstable.

Very elegant, recent work in adaptive dynamics also considers what happens if there is coexistence of more than one strategy and looks at bifurcations in the strategy space (Metz *et al*., 1996). Nevertheless, the population is assumed to consist of only one or a few different strategies at any one time.

In this paper, we will use adaptive dynamics to describe the evolutionary Ultimatum Game. The standard framework of adaptive dynamics is not applicable, because the payoff function is not differentiable at points at which the offer and acceptance levels are equal. If we restrict the strategy set to such strategies, then every strategy is a strict Nash equilibrium and the adaptive dynamics do not move at all. If we extend the framework of adaptive dynamics to consider heterogeneous populations, then we obtain meaningful evolutionary dynamics for this restricted, one-parameter Ultimatum Game. Interestingly, these evolutionary dynamics proceed towards fairness.

In Section 2, we present computer simulations for the evolutionary dynamics of the full Ultimatum Game and the one-parameter game. In Section 3, we present the standard adaptive dynamics formulation. In Section 4, we generalize the adaptive dynamics formulation and apply it to the one-parameter Ultimatum Game. In Section 5, we apply the generalized adaptive dynamics to the full Ultimatum Game and discuss the long-term dynamics. Finally, in Section 6, we present our conclusions.

## 2. The Evolutionary Dynamics of the Ultimatum Game

We consider a population of players with strategies given by $S(p, q)$. Here $p$ denotes the proportion of the sum offered by a proposer and $q$ the minimum proportion acceptable to a responder. Both $p$ and $q$ are numbers in $[0, 1]$. Initially, the strategies are uniformly distributed within the unit square $[0, 1]^2$. In each generation each player plays every other player with equal probability and the number of offspring of a player is proportional to his total payoff. We consider that in any one interaction, it is determined at random as to which player is in the role of proposer and which is in the role of responder. We compute the expected payoff. Up to a factor of $\frac{1}{2}$, which we henceforth omit, we have for the payoff that $S'(p', q')$ obtains from $S(p, q)$:

$$E(S', S) = \begin{cases} 1 - p' + p & \text{if } p \geq q', p' \geq q, \\ 1 - p' & \text{if } p < q', p' \geq q, \\ p & \text{if } p \geq q', p' < q, \\ 0 & \text{if } p < q', p' < q. \end{cases} \quad (1)$$

Offspring copy their parent's strategies with a small error in $p$ and in $q$ which is randomly distributed in an interval of size $\mu$ centered around the parental value. Numerical simulations of the evolution of such a population show that natural selection leads towards a population of players who play strategies near to the subgame perfect equilibrium $S(0, 0)$, which is the so-called rational solution (Fig. 1). In these simulations, the "mutational error" generates a diversity of strategies within the population. Thus, the mean value of $q$ remains strictly positive.

If, however, we restrict consideration to players whose offers are the same as their demands ($p = q$), which correspond to the Nash equilibria of the full Ultimatum Game, then each player's strategy is described by a single parameter $p$. Figure 2 shows the evolution of the average value of $p$ in time when $\mu = 0.01$, and there are 100 individuals in the population. In this case, the population evolves towards the "fair" strategy $S(\frac{1}{2}, \frac{1}{2})$.

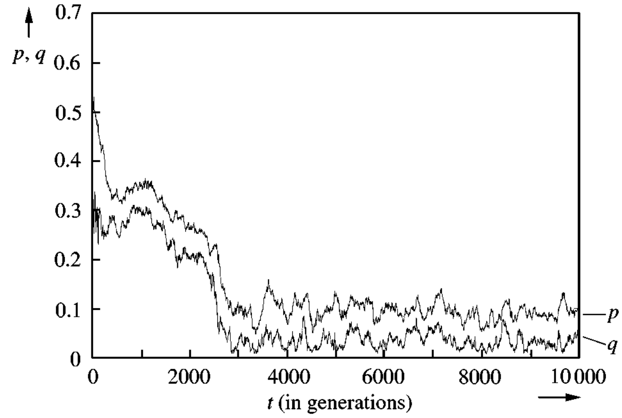


FIG. 1. The evolutionary dynamics of the Ultimatum Game converges to the "rational" strategy, $S(0, 0)$. In this computer simulation, we assume that every player interacts once in the role of proposer and once in the role of responder with every other player of the population. This deterministic payoff can also be interpreted as the average over a very large number of randomly chosen interactions. There are 100 individuals in the population. Individuals reproduce in proportion to their payoff. Offspring inherit their parent's $p$ and $q$ values plus or minus a small random mutation in the range $(-\mu/2, +\mu/2)$. In the first generation, the $p$ and $q$ values of the players are chosen randomly from the interval $[0, 1]$. We plot the mean offer, $p$, and mean acceptance level, $q$, against time for $\mu = 0.01$. The values of $p$ and $q$ do not converge exactly to the rational solution $S(0, 0)$. This is because the "mutational" error generates a diversity of strategies within the population. Thus, the mean value of $q$ remains strictly positive. Since the cost of rejection is high relative to a small increase in offer, the mean value of $p$ is close to the maximal value of $q$ in the population.

## 3. The Standard Adaptive Dynamics Formulation

The standard adaptive dynamics formulation assumes that at each time the population is homogeneous. Thus, all players adopt a single strategy in parameter space. The mutant neighborhood of this point is considered and the parameters are allowed to change in the direction which maximizes the fitness of an invading mutant when it plays a population of wild-type strategies.

Suppose the strategies can be described by some continuous parameters $p_1, \ldots, p_n$ and that the entire population plays strategy $S = S(p_1, \ldots, p_n)$. The expected payoff received by a mutant $S' = S'(p'_1, \ldots, p'_n)$ is given by $E(S', S)$. The adaptive dynamics are given by the full system of differential equations

$$\dot{p}_i = \left. \frac{\partial E(S', S)}{\partial p'_i} \right|_{S' \to S}, \quad i = 1, \ldots, n. \quad (2)$$

Hence, from eqns (1) and (2), we see that the adaptive dynamics for the Ultimatum Game are given by

$$\dot{p} = \begin{cases} -1 & \text{if } p > q, \\ 0 & \text{if } p < q, \end{cases} \quad (3)$$

$$\dot{q} = 0.$$

Thus, the adaptive dynamics tell us that for $p > q$, $p$ will decrease until the system reaches the

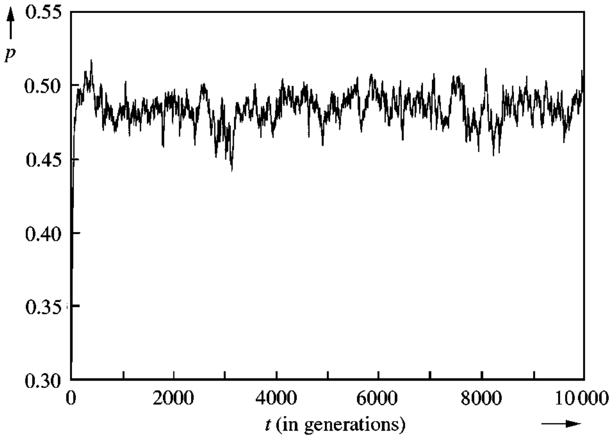


FIG. 2. Fairness evolves in the one-parameter Ultimatum Game. Players offer the minimum amount that they would be prepared to accept. Thus, their strategies are defined by the single parameter $p(=q)$. As in Fig. 1, there are 100 individuals in the population and each individual interacts with each other once in the role of proposer and once in the role of responder. Players reproduce in proportion to their payoffs and the offspring have the $p$ values of their parents plus or minus a small random mutation in the range $(-\mu/2, \mu/2)$. Initially, the players' $p$ values are chosen randomly from [0, 1]. We plot the mean value of, $p$, against time. We see that for the one-parameter game, evolution leads to a population of players offering (and thereby demanding) an almost fair split ($p=\frac{1}{2}$). The fluctuations about this value are due to randomness in the mutation error and in the selection of offspring. As before we chose $\mu = 0.01$.

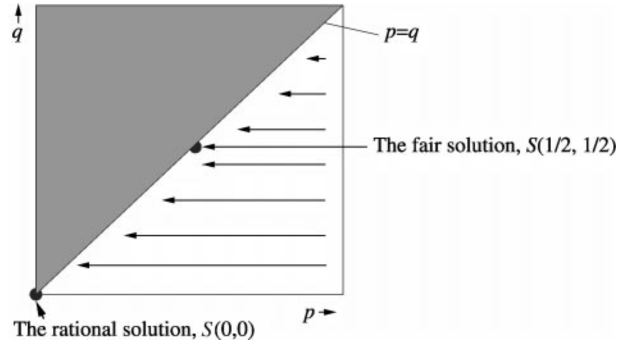


FIG. 3. Illustration of the standard adaptive dynamics of the Ultimatum Game. For $p > q$, we have $\dot{p} < 0$ and $\dot{q} = 0$, and the dynamics flow parallel to the $p$-axis, towards the line $p = q$. For $p < q$, we have $\dot{p} = \dot{q} = 0$, and all such points are equilibria for the adaptive dynamics. In this case, stochastic effects can lead to random drift. Nothing can be said about the behavior on the line $p = q$.

line of Nash equilibria $p = q$ and for $p < q$ the population will not change. For $p < q$, all offers get rejected. Thus, all nearby mutants of a strategy $S(p, q)$ are neutral variants. In a realistic setting, there will be a random drift. For a schematic illustration of the standard adaptive dynamics of the Ultimatum Game, see Fig. 3.

Adaptive dynamics cannot, however, address the behavior of the system once the line $p = q$ is reached and hence can neither describe the long-term evolution of the full Ultimatum Game nor the evolution of the one-parameter Ultimatum Game. In the next section, we develop a generalized adaptive dynamics framework and analyze the dynamics of the one-parameter game. We then use this framework to address the long-term evolution of the full Ultimatum Game. We shall see that this framework correctly captures the convergence towards the rational solution in the full Ultimatum Game and towards the fair solution in the one-parameter game.

## 4. Adaptive Dynamics of the One-parameter Ultimatum Game

The strategies of players are described by a single parameter $p$, which is both the offer that the player makes and the minimum offer that he is prepared to accept. We denote the strategy of a player with $p = q = p_i$ by $S_i$. The expected payoff that a player with strategy $S_2$ receives from a player with strategy $S_1$ is given by

$$E(S_2, S_1) = (1-p_2)H(p_2 - p_1) + p_1 H(p_1 - p_2), \qquad (4)$$

where $H(\cdots)$ denotes the Heaviside function, which takes value 1 if its argument is positive and value 0 if it is negative.

Now, suppose that the population is almost homogeneous with offers uniformly distributed within a small neighborhood of $p([p-\varepsilon/2, p+\varepsilon/2])$. The average payoff received by a player with strategy $S_2$ $(p_2 \in [p-\varepsilon/2, p+\varepsilon/2])$, is given by

$$E(S_2, \bar{S}) = \frac{1}{\varepsilon}\int_{p-\varepsilon/2}^{p+\varepsilon/2} E(S_2, S_1)\,\mathrm{d}p_1. \qquad (5)$$

Differentiating with respect to $p_2$, at $p_2 = p$, we find

$$\left.\frac{\partial E(S_2, \bar{S})}{\partial p_2}\right|_{p_2=p} = \frac{1}{\varepsilon}\frac{\partial}{\partial p_2}\left.\left(\int_{p-\varepsilon/2}^{p+\varepsilon/2} E(S_2, S_1)\, \mathrm{d}p_1\right)\right|_{p_2=p}$$

$$\equiv \frac{1}{\varepsilon}\int_{p-\varepsilon/2}^{p+\varepsilon/2} \left.\frac{\partial}{\partial p_2} E(S_2, S_1)\right|_{p_2=p} \mathrm{d}p_1 \quad (6)$$

$$= \frac{1}{\varepsilon}\int_{p-\varepsilon/2}^{p+\varepsilon/2} - H(p-p_1) + (1-2p)\delta(p-p_1)\, \mathrm{d}p_1$$

$$= -\frac{1}{2} + \frac{(1-2p)}{\varepsilon}. \quad (7)$$

Here $\delta(\cdots)$ is the Dirac delta function. Thus, following the standard adaptive dynamics formulation, in which the average strategy in the population moves in the direction which maximizes its payoff against the resident strategy, we have, for the time derivative of $p$,

$$\dot{p} = \frac{(1-2p)}{\varepsilon} - \frac{1}{2}. \quad (8)$$

Rescaling time, we obtain

$$\dot{p} = 1 - 2p - \frac{\varepsilon}{2}. \quad (9)$$

Thus, the generalized adaptive dynamics predict that offers will converge to $p \approx \frac{1}{2}$ (for small $\varepsilon$) in the one-parameter Ultimatum Game. This prediction is supported by numerical simulations, see Fig. 2.

## 5. Generalized Adaptive Dynamics of the Full Ultimatum Game

In the full Ultimatum Game the strategies are described by an offer, $p$ and an independent aspiration level, $q$, which is the minimum offer acceptable to the player. If we again assume that the population is almost homogeneous and that strategies are uniformly distributed in $[p-\varepsilon/2, p+\varepsilon/2] \times [q-\varepsilon/2, q+\varepsilon/2]$, then the expected payoff to a strategy $S_2 = (p_2, q_2)$ is given by

$$E(S_2, \bar{S}) = \frac{1}{\varepsilon^2}\int_{p-\varepsilon/2}^{p+\varepsilon/2}\int_{q-\varepsilon/2}^{q+\varepsilon/2} (1-p_2)H(p_2-q_1) + p_1 H(p_1-q_2)\, \mathrm{d}q_1\, \mathrm{d}p_1. \quad (10)$$

Differentiating, we find that

$$\frac{\partial}{\partial p_2} E(S_2, \bar{S})|_{S_2\to S} = \frac{1}{\varepsilon}\int_{q-\varepsilon/2}^{q+\varepsilon/2} - H(p-q_1) + (1-p)\delta(p-q_1)\, \mathrm{d}q_1, \quad (11)$$

$$\frac{\partial}{\partial q_2} E(S_2, \bar{S})|_{S_2\to S} = \frac{1}{\varepsilon}\int_{p-\varepsilon/2}^{p+\varepsilon/2} -q\delta(p_1-q)\, \mathrm{d}p_1. \quad (12)$$

We have to distinguish three cases:
(i) For $p > q + \varepsilon/2$, we have

$$\frac{\partial}{\partial p_2} E(S_2, \bar{S})|_{S_2\to S} = -1, \qquad \frac{\partial}{\partial q_2} E(S_2, \bar{S})|_{S_2\to S} = 0. \quad (13)$$

(ii) For $p \in [q-\varepsilon/2, q+\varepsilon/2]$, we have

$$\frac{\partial}{\partial p_2} E(S_2, \bar{S})|_{S_2\to S} = \frac{1-p}{\varepsilon} - \frac{1}{\varepsilon}(p-q+\varepsilon/2), \qquad \frac{\partial}{\partial q_2} E(S_2, \bar{S})|_{S_2\to S} = -\frac{q}{\varepsilon}. \quad (14)$$

(iii) For $p < q - \varepsilon/2$, we have

$$\frac{\partial}{\partial p_2} E(S_2, \bar{S})|_{S_2\to S} = 0, \qquad \frac{\partial}{\partial q_2} E(S_2, \bar{S})|_{S_2\to S} = 0. \quad (15)$$

Thus, our generalized adaptive dynamics yield, after a change of timescales,

$$\dot{p} = \begin{cases} -1, & \text{if } p > q + \varepsilon/2, \\ 1 - p + O(\varepsilon), & \text{if } p \in [q - \varepsilon/2, q + \varepsilon/2], \\ 0 & \text{if } p < q - \varepsilon/2. \end{cases}$$

$$\dot{q} = \begin{cases} 0 & \text{if } p > q + \varepsilon/2, \\ -q & \text{if } p \in [q - \varepsilon/2, q + \varepsilon/2], \\ 0 & \text{if } p < q - \varepsilon/2. \end{cases} \quad (16)$$

When the average strategy in the population satisfies $q < p - \varepsilon/2$, then $p$ will decrease and $q$ will remain constant. Thus, ultimately the average strategy will enter the region $p \in [q - \varepsilon/2, q + \varepsilon/2]$, but according to the adaptive dynamics it will immediately exit again. The average strategy will hence converge close to the line $p = q + \varepsilon/2$. For non-zero $\varepsilon$, the changes in the average offer and acceptance level will be non-zero (although small) and since the value of $q$ decreases when $p \in [q - \varepsilon/2, q + \varepsilon/2]$, and remains constant in $p > q + \varepsilon/2$, $q$ will decrease in time and the average strategy will approach (0, 0) (for $\varepsilon$ very small). This agrees with numerical simulations, which show that, when replication is accurate (small $\mu$), natural selection favors the unfair, rational solution (Fig. 1). We illustrate the generalized adaptive dynamics schematically in Fig. 4.

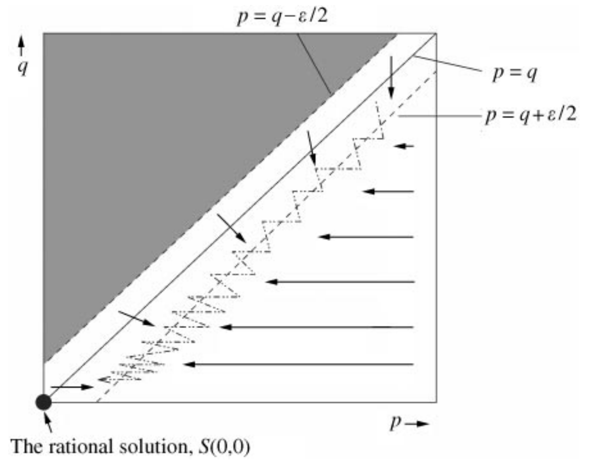


FIG. 4. Illustration of the generalized adaptive dynamics of the Ultimatum Game. For $|p - q| > \varepsilon/2$, the dynamical flow is the same as in Fig. 3. For $p > q + \varepsilon/2$, we have $\dot{p} < 0$ and $\dot{q} = 0$, and the dynamics flow parallel to the $p$-axis, towards the line $p = q$. For $p < q - \varepsilon/2$, we have $\dot{p} = \dot{q} = 0$, and all such points are equilibria for the adaptive dynamics. For $p \in [q - \varepsilon/2, q + \varepsilon/2]$, the flow is towards the line $p = q + \varepsilon/2$ in the direction of increasing $p$ and decreasing $q$. Since the changes in $p$ and $q$ between generations are small but non-zero, the trajectory of the average strategy $(p, q)$ will repeatedly cross this line $p = q + \varepsilon/2$ and the dynamics will flow towards (0, 0), as illustrated in the figure.

## 6. Conclusions

We have extended the framework of adaptive dynamics to apply to the games in which the payoffs are not continuous in parameter space. By assuming that the population is distributed around an average strategy rather than being completely homogeneous, and computing the average payoff obtained by a mutant, it is possible to determine the dynamics at a point of discontinuity of the payoff. In the above analysis, we have assumed that the population is uniformly distributed around the average strategy. However, all the results hold likewise for other distributions.

Being able to determine the dynamics at a point of discontinuity of the payoff is important in the context of the Ultimatum Game, since all the Nash equilibria are on the line $p = q$. This is a line of discontinuities of the payoffs, where standard adaptive dynamics is not applicable and so cannot be used to address the long-term dynamics.

We find that the adaptive dynamics of the full Ultimatum Game lead to the rational solution, where players offer next to nothing. In the one-parameter Ultimatum Game, however, the adaptive dynamics predict the evolution of fairness ($p = q = \frac{1}{2}$). Both predictions are confirmed by detailed computer simulations (Figs 1 and 2).

Playing $p = q$ is equivalent to assuming that the responder follows a strategy identical to one's own strategy. In this case, the best option is to offer one's own $q$ value. Hence, by using one's own strategy as a model for the opponent (which is a kind of "theory of mind" approach), the game is reduced to one parameter.

Interestingly, evolution also leads to fairness if only a small fraction of players offer their acceptance level, $q$. This will be the topic of a subsequent paper (Page & Nowak, in preparation).

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