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Indirect reciprocity with optional interactions

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HIGHLIGHTS

- We study indirect reciprocity with optional interactions.
- Players can decline a game based on the reputation of the co-player.
- The payoff function is nonlinear in frequency.
- We calculate conditions for evolution of cooperation.
- We also study effect of hesitation and malicious gossip.

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ABSTRACT

Indirect reciprocity is a mechanism for the evolution of cooperation that is relevant for prosocial behavior among humans. Indirect reciprocity means that my behavior towards you also depends on what you have done to others. Indirect reciprocity is associated with the evolution of social intelligence and human language. Most approaches to indirect reciprocity assume obligatory interactions, but here we explore optional interactions. In any one round a game between two players is offered. A cooperator accepts a game unless the reputation of the other player indicates a defector. For a game to take place, both players must accept. In a game between a cooperator and a defector, the reputation of the defector is revealed to all players with probability Q. After a sufficiently large number of rounds the identity of all defectors is known and cooperators are no longer exploited. The crucial condition for evolution of cooperation can be written as $hQB > 1$, where h is the average number of rounds per person and $B = (b/c) - 1$ specifies the benefit-to-cost ratio. We analyze both stochastic and deterministic evolutionary game dynamics. We study two extensions that deal with uncertainty: hesitation and malicious gossip.

1. Introduction

Prosocial behavior is a key aspect of human interaction. We sometimes help others although it entails a cost for ourselves. The efficiency of human society and its evolution are based to a large extent on cooperation. Cooperation is a conundrum in biology. Why should we help a competitor in the struggle for survival? Why should we contribute to a public good if free riders reap the rewards of our generosity? Natural selection opposes cooperation unless a mechanism for the evolution of cooperation is in place. Direct reciprocity, indirect reciprocity, spatial selection, group selection and kin selection represent five mechanisms for the evolution of cooperation (Nowak, 2006a). All five mechanisms shape human cooperation, but direct and indirect reciprocity play a central role, because most of our crucial interactions occur in the context of repetition and reputation (Rand and Nowak, 2013).

Direct reciprocity is based on repeated interactions between the same two players: my behavior towards you depends on what has happened between us in previous encounters (Rapoport and Chammah, 1965; Trivers, 1971; Axelrod, 1984; Molander, 1985; Fudenberg and Maskin, 1986; May, 1987; Milinski, 1987; Kraines and Kraines, 1989; Nowak and Sigmund, 1989, 1992, 1993, 1994; Fudenberg and Maskin, 1990; Boerlijst et al., 1997; Imhof et al., 2005; Dreber et al., 2008; Van Veelen et al., 2012). Indirect reciprocity is based on repeated interactions in a group of players; my behavior toward you also depends on what you have done to others (Nowak and Sigmund, 1998a,b, 2005, Leimar and Hammerstein, 2001; Fishman, 2003; Panchanathan and Boyd,

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2003, 2004; Brandt and Sigmund, 2004, 2005, 2006; Ohtsuki and Iwasa, 2004, 2006, 2007; Takahashi and Mashima, 2006; Suzuki and Akiyama, 2007a,b; Roberts, 2008; Ohtsuki et al., 2009; Sigmund, 2010, 2012; Uchida, 2010; Uchida and Sigmund, 2010; Berger, 2011; Nakamura and Masuda, 2011; Martinez-Vaquero and Cuesta, 2013; Suzuki and Kimura, 2013; Tanabe et al., 2013; Berger and Grüne, 2014; Jusup et al., 2014; Matsuo et al., 2014). Indirect reciprocity arises from direct reciprocity in the context of an interested audience (Sugden, 1986). Interactions between people are observed by third parties. Information spreads through gossip. People take a keen interest in who does what to whom and why. The games of indirect reciprocity provide a selection pressure for both social intelligence and human language (Nowak and Sigmund, 2005). Indirect reciprocity is a major factor for the emergence of moral systems (Alexander, 1987).

Indirect reciprocity works via reputation (Nowak and Sigmund, 1998a,b). Cooperation in the right context leads to a good reputation. Defection can lead to a bad reputation. Natural selection favors the evolution of strategies that base their decision whether or not to cooperate on the reputation of the recipient. Numerous experiments show that those who are helpful to others are more likely to receive help (Wedekind and Milinski, 2000; Dufwenberg et al., 2001; Milinski et al., 2002a,b; Wedekind and Braithwaite, 2002; Bolton et al., 2004, 2005; Seinen and Schram, 2005; Bshary and Grutter, 2006; Rockenbach and Milinski, 2006; Milinski and Rockenbach, 2007; Sommerfeld et al., 2007; Bshary et al., 2008; Engelmann and Fischbacher, 2009; Ernest-Jones et al., 2011; Jacquet et al., 2012; Yoeli et al., 2013). Helpful people have a higher payoff in the end.

A strategy for playing indirect reciprocity consists of an action rule and a social norm (Nowak and Sigmund, 2005). The action rule specifies whether or not to cooperate given one's own reputation and that of the recipient. The social norm determines how to interpret the actions of others. A simple social norm is 'image scoring': cooperation is good and defection is bad. But a problem arises when a cooperator meets a defector: the defection that is now warranted can damage the reputation of the cooperator. A more sophisticated social norm is 'standing': defection with an undeserving partner does not reduce one's own reputation (Sugden, 1986). Another norm is 'judging': cooperation with an undeserving partner reduces one's own reputation (Kandori, 1992). Both standing and judging are vulnerable to deception: a typical human strategy is to tarnish the reputation of another person in order to justify one's own defection.

Most approaches to indirect reciprocity use obligatory interactions, where engagement with defectors cannot be avoided. But in this paper we open a new avenue toward examining indirect reciprocity by studying optional interactions. A game is offered between two individuals. The game takes place if both players accept. Cooperators accept unless it is known that the other player has the reputation of a defector. The reputation of a defector is revealed – with a certain probability – if that defector (still unknown) manages to exploit a cooperator. This optional approach has the advantage of avoiding much of the strategic complexity that is necessary for obligatory games of indirect reciprocity. Cooperators need not defect when meeting a defector and thereby possibly harm their own reputation, but instead they can refuse to play the game. This setup is natural. On the web we avoid buying from sellers with dubious reputation instead of seeking them out and defecting with them. In daily life we also avoid interaction with defectors if this is possible.

Refusing interaction with defectors can also be seen as a form of partner choice (Nöe and Hammerstein, 1994; Pacheco et al., 2006b; Fu et al., 2008). There have been numerous studies of evolutionary games in the context of optional interactions and partner choice (Hauert et al., 2002, 2007). Sometimes they are described in terms of dynamical graphs (Wardil and Hauert, 2014). If you end an interaction because a player has defected against you, then this is a form of direct reciprocity (Pacheco et al., 2006a). If you end or do not begin an interaction, because a player has defected against someone else, then this is a form of indirect reciprocity (Fu et al., 2008). Likewise it is a form of indirect reciprocity if you preferentially seek interactions with people who have cooperated with others (Tarnita et al., 2009; Du and Fu, 2011; Cavaliere et al., 2012). Optional interactions and ostracism have also been discussed in the context of multi-player games such as the public goods game or common pool resource games (Aktipis, 2004; Chiang, 2008; Nakamura and Masuda, 2012; Tavoni et al., 2012; Lade et al., 2013).

The paper is structured as follows. In Section 2 we introduce our basic model and give our main results. In Section 3 we provide derivations. In Section 4 we extend our basic model by assuming that cooperators have a hesitation to interact, because they are uncertain about the reputation of the other individual. In Section 5 we study malicious gossip: sometimes cooperators are wrongly accused as defectors. Section 6 offers conclusions.

2. Basic model and main results

We study an optional game in a population of N individuals. There are multiple rounds. In any one round two random players are chosen, and a game is offered between them. The game takes place if both accept to play. In the next round two (other) random players are chosen and so on. There is a probability w to continue the game after each round. The average number of rounds is $M = 1 + w + w^2 + \cdots = 1/(1-w)$. Consequently the average number of potential games offered to each player is $h = 2M/N$. Throughout the paper we consider the case $M > 1$ and therefore we also have $h > 2/N$.

There are two types of players, cooperators and defectors. Cooperators pay a cost, c, for the other player to receive a benefit, b, where $b > c > 0$. Defectors pay no cost and provide no benefit. We assume that a cooperator accepts to play a game as long as the other player is unknown or known to be a cooperator. A defector always accepts to play the game. In the beginning of the game all players are unknown. If a game occurs where one individual is a cooperator and the other is a defector, then the reputation of the defector is established in the population with probability Q. If a player is known to be a defector, then in all subsequent rounds cooperators refuse to play with him. A game between two defectors leads to zero payoff for both players and is of no consequence with respect to payoff; neither of the two players is exploited. In this paper, we assume that in a game between two defectors no reputation is established or revealed. This assumption helps us to mask the identity of defectors and leads to more stringent conditions for evolution of cooperation. The other possibility is that even in a game between two defectors the reputation of each one is revealed with a certain probability. In this case defectors can lose their anonymity even when interacting with each other and without exploiting cooperators. Therefore the conditions for evolution of cooperation become more lenient. We will explore this option, which requires more complicated calculations, in a subsequent paper. We now state our main results. The derivations are given in the next section.

2.1. Finite population size

At first we present a Nash equilibrium type argument. If everyone in the population is a cooperator, the expected payoff for each player is

$$F_N = (b-c)h \tag{1}$$

Every potential game is accepted. On average there are h many games per player and each game results in a payoff $b-c$. We use the following notation: F_i is the expected payoff for a cooperator if there are i many cooperators in the population; G_i is the expected payoff for a defector if there are i many cooperators in the population.

Now let us calculate the incentive to deviate from cooperation. In particular, we need to calculate the expected payoff for a single defector in a group with $N-1$ cooperators. We obtain

$$G_{N-1} = \frac{bh}{1+hQ-\frac{2Q}{N}} \tag{2}$$

For each accepted game the defector receives payoff b. If $Q=1$ the defector can play at most one game. The condition $F_N > G_{N-1}$ can be written as

$$h > \frac{1}{BQ} + \frac{2}{N} \tag{3}$$

Here we use the notation $B=(b/c)-1$. If inequality (3) holds, then cooperation is a strict Nash equilibrium. Therefore, we need a minimum average number of rounds per player. This number depends inversely on the benefit-to-cost ratio and on the probability that the identity of a defector is revealed. There is an additive term, $2/N$, which becomes small for large population size. In the limit of large population size, we obtain the simple condition for cooperation to be a strict Nash equilibrium:

$$hQB > 1 \tag{4}$$

More generally, we calculate the average payoffs if there are i cooperators and $N-i$ defectors. In any one round, the probability that a game is offered between two cooperators is $p=\binom{i}{2}/\binom{N}{2}$; the probability that a game is offered between a cooperator and a defector is $q=i(N-i)/\binom{N}{2}$. For the average payoff of a cooperator we derive

$$F_i = \frac{1}{i}\left(\frac{2(b-c)p}{1-w} - \frac{cq}{1-\left(1-\frac{Qq}{N-i}\right)w}\right) \tag{5}$$

For the average payoff of a defector we derive

$$G_i = \frac{bq}{(N-i)\left[1-\left(1-\frac{Qq}{N-i}\right)w\right]} \tag{6}$$

Using $w=1-2/(Nh)$ we obtain

$$F_i = \frac{Nh}{i}\left((b-c)p - \frac{cq(N-i)}{Qq(Nh-2)+2(N-i)}\right) \tag{7}$$

and

$$G_i = \frac{Nhbq}{Qq(Nh-2)+2(N-i)} \tag{8}$$

We make the following observations: (i) $F_1 < G_1$; a single cooperator in a population of defectors always has a lower average payoff than the defectors. (ii) The sign of $F_i - G_i$ changes at most once as i increases. Thus, if there is an index j where $F_j > G_j$, then $F_i > G_i$ for all $i > j$. (iii) $F_{N-1} > G_{N-1}$, which means the cooperators have a higher payoff than the defector if there is a single defector, is equivalent to

$$h > \frac{N+B}{BQ(N-2)} + \frac{2}{N} \tag{9}$$

This condition is slightly more restrictive than the criterion of cooperation being a strict Nash equilibrium (3), but also converges to $hBQ > 1$ for large N.

We can also ask whether cooperation is stable against a coalition of k players who switch simultaneously to defection. The condition $F_N > G_{N-k}$ with $k<N$ leads to

$$h > \frac{1}{BQ}\left(1 - B\frac{k-1}{N-k}\right) + \frac{2}{N} \tag{10}$$

The condition for cooperation to be a strict Nash equilibrium (3) implies stability against a coalition of k players switching to defection.

We can use Eqs. (5) and (6) to formulate game dynamics in finite populations (Nowak et al., 2004; Taylor et al., 2004; Imhof and Nowak, 2006; Antal et al., 2009a). In particular we calculate the fixation probabilities ρ_C for a single cooperator and ρ_D for a single defector. For low mutation, selection favors cooperation over defection if $\rho_C > \rho_D$. The payoff, however, is a nonlinear function of i and therefore we may not obtain simple conditions. In the limit of weak selection and for large population size, we find that $\rho_C > \rho_D$ is equivalent to

$$BH(H-2) + 2(B-H)\log(1+H) > 0 \tag{11}$$

Here we have used the notation $H=hQ$. Condition (11) is easier fulfilled for larger B and for larger H, which is a consistency check. Also note that (11) implies $HB > 1$. Therefore, $hBQ > 1$, is a necessary condition for $\rho_C > \rho_D$ in the limit of weak selection and large population size.

2.2. Deterministic dynamics, infinite population size

We can study deterministic evolutionary dynamics in the limit $N \to \infty$ while fixing h, which is the average number of optional rounds offered to each player. The frequency of cooperators is $x=i/N$. The frequency of defectors is $1-x$. For the average payoff of cooperators we derive

$$F(x) = h(b-c)x - \frac{ch(1-x)}{hQx+1} \tag{12}$$

$$G(x) = \frac{bhx}{hQx+1} \tag{13}$$

Frequency dependent selection dynamics can be described by the following differential equation:

$$\dot{x} = x(1-x)[F(x) - G(x)] \tag{14}$$

We obtain

$$\dot{x} = x(1-x)\frac{HBx^2-1}{Hx+1} \tag{15}$$

This is a replicator equation with a nonlinear fitness function. We have omitted a factor ch which only rescales time. Again we use the convenient notation $H=hQ$ and $B=(b/c)-1$.

The dynamical behavior can be characterized as follows. First, note that the all-defect equilibrium, $x=0$, is always stable. If $HB<1$, then the all-cooperate equilibrium, $x=1$, is unstable; in this case cooperation is dominated by defection. If on the other hand $HB>1$, then the all-cooperate equilibrium is also stable, and we obtain bi-stability. There is a single unstable equilibrium in the interior, which is given by

$$x^* = \frac{1}{\sqrt{HB}}. \tag{16}$$

If the initial frequency of cooperators is greater than x^* then the selection dynamics will converge to $x=1$; otherwise convergence is to $x=0$.

We note that the crucial condition, $HB>1$, is equivalent to the strict Nash condition (3) in the limit of large N, as expected. There is also a simple condition for cooperators to have a bigger basin of attraction than defectors: $HB>4$. In this case, cooperation is 'risk dominant' over defection in the sense that $x^* < 1/2$ which means that if we start with a uniformly randomly distributed frequency of cooperators on the interval [0, 1], then the selection dynamics is more likely to converge to the all-cooperate equilibrium, $x=1$, than to the all-defect equilibrium, $x=0$.

3. Derivation of results

We first calculate the expected payoff, then derive stochastic evolutionary dynamics in populations of finite size, and finally derive deterministic evolutionary dynamics in populations of infinite size.

3.1. Calculation of payoff

Consider a population of size N with i cooperators and $N-i$ defectors. We first calculate the payoffs after exactly m rounds. Denote the expected payoff for each cooperator after m rounds by $F_i(m)$ and the expected payoff for each defector by $G_i(m)$. The index i denotes the number of cooperators in the population.

There are three types of games: CC, CD, DD. For each round the probability to have a CC game is $p=\binom{i}{2}/\binom{N}{2}$. The probability to have a CD game is $q=i(N-i)/\binom{N}{2}$. The probability to have a DD game is $r=\binom{N-i}{2}/\binom{N}{2}$. Therefore the probability that there are exactly j many CC games, k many CD games and l many DD games after m rounds is given by $\binom{m}{j,k,l}p^jq^kr^l$. We have the constraint $j+k+l=m$.

Each CC game brings a payoff $b-c$ to two cooperators. DD games have no effect. The effect of a CD game depends on whether the identity of the defector is already known or not. If the defector is known, then the game will not take place and there is no further consequence. If the identity of the defector is unknown, then the game will take place and the cooperator is exploited; the cooperator receives payoff $-c$ while the defector receives payoff b. Moreover there is a probability Q that the identity of the defector is revealed after the game. Hence we need to find the expected number of exploitative CD games given that k many CD games were offered in total. Let us denote this number by e_k.

We now compute e_k. Let us label the $N-i$ defectors by $D_1, D_2, \ldots, D_{N-i}$. Given that there are k many optional CD interactions, the probability that D_1 is offered k_1 games with cooperators, D_2 is offered k_2 games with cooperators,…, and D_{N-i} is offered k_{N-i} games with cooperators is $\binom{k}{k_1,k_2,\ldots,k_{N-i}}/(N-i)^k$ for nonnegative integers $k_1, k_2, \ldots, k_{N-i}$ such that $k_1+k_2+\cdots+k_{N-i}=k$.

Suppose that a particular defector has been offered k_j many games with cooperators. In the first game the defector always exploits the cooperator. In the second game, the defector can exploit iff his identity was not revealed. Therefore the defector exploits the cooperator in the second game with the probability $(1-Q)$. Similarly, the defector exploits the cooperator in the k_jth game iff his identity has not been revealed in all preceding games. This happens with probability $(1-Q)^{k_j-1}$. Hence the expected number of exploitations done by a defector, who has been offered k_j games with cooperators, is

$$1+(1-Q)+(1-Q)^2+\cdots+(1-Q)^{k_j-1}=\frac{1-(1-Q)^{k_j}}{Q}. \tag{17}$$

Now we can explicitly calculate e_k, the expected number of exploitations done by defectors. We have

$$\begin{aligned}
e_k &= \sum_{k_1+\cdots+k_{N-i}=k}\left(\binom{k}{k_1,k_2,\ldots,k_{N-i}}\Big/(N-i)^k\right)\sum_{j=1}^{N-i}\frac{1-(1-Q)^{k_j}}{Q}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1+\cdots+k_{N-i}=k}\binom{k}{k_1,k_2,\ldots,k_{N-i}}(1-(1-Q)^{k_1})\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(1-(1-Q)^{k_1})\sum_{k_2+\cdots+k_{N-i}=k-k_1}\binom{k-k_1}{k_2,k_3,\ldots,k_{N-i}}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(1-(1-Q)^{k_1})(N-i-1)^{k-k_1}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(N-i-1)^{k-k_1}-(1-Q)^{k_1}(N-i-1)^{k-k_1}\\
&=\frac{N-i}{Q(N-i)^k}[(N-i)^k-(N-i-Q)^k]\\
&=\frac{N-i}{Q}\left[1-\left(1-\frac{Q}{N-i}\right)^k\right]
\end{aligned} \tag{18}$$

Therefore the expected payoff for each cooperator after m rounds is

$$\begin{aligned}
F_i(m) &= \frac{1}{i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^l(2j(b-c)-ce_k)\\
&=\frac{1}{i}\left(2m(b-c)p-\frac{c(N-i)}{Q}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^l\left(1-\left(1-\frac{Q}{N-i}\right)^k\right)\right)\\
&=\frac{1}{i}\left(2m(b-c)p-\frac{c(N-i)}{Q}\left(1-\left(p+q\left(1-\frac{Q}{N-i}\right)+r\right)^m\right)\right)\\
&=\frac{1}{i}\left(2m(b-c)p-\frac{c(N-i)}{Q}\left(1-\left(1-\frac{qQ}{N-i}\right)^m\right)\right)
\end{aligned} \tag{19}$$

Similarly, the expected payoff for each defector after m rounds is

$$\begin{aligned}
G_i(m) &= \frac{1}{N-i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^lbe_k\\
&=\frac{1}{N-i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^lb\frac{N-i}{Q}\left(1-\left(1-\frac{Q}{N-i}\right)^k\right)\\
&=\frac{b}{Q}\left(1-\left(1-\frac{qQ}{N-i}\right)^m\right)
\end{aligned} \tag{20}$$

Note that in order to have exactly m rounds, the game has to continue for the first $(m-1)$ rounds and then stop at round m. This happens with probability $(1-w)w^{m-1}$. Therefore the expected payoff for each cooperator for the whole game is

$$\begin{aligned}
F_i &= \sum_{m=1}^{\infty}(1-w)w^{m-1}F_i(m)\\
&=\frac{(1-w)}{i}\sum_{m=1}^{\infty}w^{m-1}\left(2mp(b-c)-\frac{c(N-i)}{Q}\left(1-\left(1-\frac{qQ}{N-i}\right)^m\right)\right)\\
&=\frac{(1-w)}{i}\left(\frac{2(b-c)p}{(1-w)^2}-\frac{c(N-i)}{Q}\left(\frac{1}{1-w}-\frac{1-\frac{qQ}{N-i}}{1-\left(1-\frac{qQ}{N-i}\right)w}\right)\right)\\
&=\frac{(1-w)}{i}\left(\frac{2(b-c)p}{(1-w)^2}-\frac{c(N-i)}{Q}\frac{\frac{qQ}{N-i}}{(1-w)\left(1-\left(1-\frac{qQ}{N-i}\right)w\right)}\right)\\
&=\frac{1}{i}\left(\frac{2(b-c)p}{1-w}-\frac{cq}{1-\left(1-\frac{qQ}{N-i}\right)w}\right).
\end{aligned} \tag{21}$$

Here we have used the formula for the geometric series

$$\frac{1}{1-x}=\sum_{m=1}^{\infty}x^{m-1} \tag{22}$$

and another formula which comes from taking derivative of the above equation:

$$\frac{1}{(1-x)^2}=\sum_{m=1}^{\infty}mx^{m-1} \tag{23}$$

Similarly, we can find the expected payoff for each defector for the whole game:

$$\begin{aligned} G_i &= \sum_{m=1}^{\infty} (1-w)w^{m-1} G_i(m) \\ &= \frac{b(1-w)}{Q} \sum_{m=1}^{\infty} w^{m-1} \left(1 - \left(1 - \frac{qQ}{N-i}\right)^m\right) \\ &= \frac{b(1-w)}{Q} \left(\frac{1}{1-w} - \frac{1 - \frac{qQ}{N-i}}{1 - \left(1 - \frac{qQ}{N-i}\right) w} \right) \\ &= \frac{b(1-w)}{Q} \left(\frac{\frac{qQ}{N-i}}{(1-w)\left(1 - \left(1 - \frac{qQ}{N-i}\right) w\right)} \right) \\ &= \frac{bq}{(N-i)\left(1 - \left(1 - \frac{qQ}{N-i}\right) w\right)} \end{aligned} \tag{24}$$

For continuation probability w, the average number of rounds is given by $M = 1/(1-w)$. The average number of rounds offered to each player is $h = 2M/N$. Therefore we have $w = 1 - 2/(hN)$. It is convenient to use the payoff parameter $B = (b/c) - 1$. Together with the relations $p = \binom{i}{2}/\binom{N}{2}$ and $q = i(N-i)/\binom{N}{2}$, we can express F_i and G_i as

$$\begin{aligned} F_i &= ch \left(\frac{B(i-1)}{N-1} - \frac{N-i}{N-1-2Q\frac{i}{N} + hQi} \right) \\ G_i &= ch \frac{(B+1)i}{N-1-2Q\frac{i}{N} + hQi} \end{aligned} \tag{25}$$

Let us now discuss some properties of this payoff function. First we note that everyone being a defector is always a strict Nash equilibrium. The expected payoff for a single cooperator in a population of defectors is negative, $F_1 < 0$, while the payoff for defectors in an all-defector population is $G_0 = 0$.

Next we investigate the condition for cooperation to be a strict Nash equilibrium. If everyone is a cooperator the payoff is $F_N = chB$. If one player switches to defection, his payoff becomes

$$G_{N-1} = \frac{ch(B+1)}{1 - \frac{2Q}{N} + hQ} \tag{26}$$

From $F_N > G_{N-1}$ we derive

$$h > \frac{1}{BQ} + \frac{2}{N} \tag{27}$$

Finally we investigate the sign of $F_i - G_i$. The sign of this expression is the same as the sign of $i(i-1)QB(h-2/N) - (N+B)(N-1)$, which is an increasing function of i. For every h, the sign is negative for $i=1$, and it has at most one sign change as i increases from 1 to $N-1$. Therefore, if $F_j < G_j$ for some j then $F_i < G_i$ for all $i = 1, \dots, j-1$. These features are illustrated in Fig. 1. The propensity for indirect reciprocity to promote cooperators over defectors works best in populations with few defectors and becomes worse as the number of defectors increases.

3.2. Stochastic game dynamics for finite population size

We now study stochastic evolutionary game dynamics in a population of finite size (Nowak et al., 2004; Taylor et al., 2004; Imhof and Nowak, 2006, 2010; Nowak, 2006b; Lessard and Ladret, 2007; Traulsen et al., 2007; Antal et al., 2009a,b; Altrock et al., 2012; Wu et al., 2013). We consider a frequency dependent Moran process (Moran, 1962). At any one time step a random individual is chosen for reproduction proportional to its fitness and a random individual is chosen for death. The total population size remains strictly constant. There are i cooperators and $N-i$ defectors. The state space of the stochastic process is given by $i = 0, 1, \dots, N$. The states $i = 1, \dots, N-1$ are transient. The states $i=0$ and $i=N$ are absorbing. The process is a birth–death process, because at any one time the state i can at most change by ± 1.

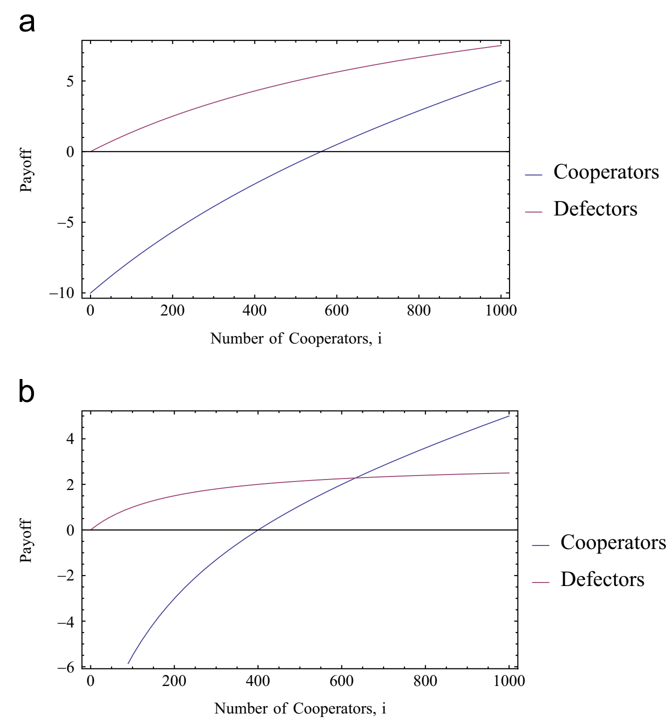

Fig. 1. The payoff values F_i and G_i of cooperators and defectors are nonlinear functions of the number of cooperators, i. There are two possibilities. Either defectors always have a higher payoff, which means $G_i > F_i$ for all i, or there is a cross over point. In the later case we have $G_i > F_i$ for small i, but $G_i < F_i$ for large i. The difference between the two panels is the identification probability, Q. (a) We have $Q=0.1$ and (b) $Q=0.5$. All other parameters are the same: $N=1000$, $h=10$, $b=1.5$ and $c=1$; which means $B = (b/c) - 1 = 0.5$. Note that for (a) we have $hBQ=0.5$ which is less than one and thus we expect that cooperation is not favored. For (b) we have $hBQ=2.5$ which is greater than one and thus cooperation may be favored.

If there are i cooperators then the fitness of each cooperator is $f_i = 1 + \delta F_i$, while the fitness of each defector is $g_i = 1 + \delta G_i$. The parameter δ denotes the intensity of selection. The limit of weak selection is given by $\delta \to 0$.

The fixation probability of a single cooperator is

$$\rho_C = 1 \bigg/ \left[1 + \sum_{k=1}^{N-1} \prod_{i=1}^{k} \frac{g_i}{f_i} \right] \tag{28}$$

The fixation probability of a single defector is

$$\rho_D = \prod_{i=1}^{N-1} \frac{g_i}{f_i} \bigg/ \left[1 + \sum_{k=1}^{N-1} \prod_{i=1}^{k} \frac{g_i}{f_i} \right] \tag{29}$$

If $\rho_C > 1/N$ then the fixation of cooperators is favored by selection. If $\rho_D < 1/N$ then the fixation of defectors is opposed by selection. If $\rho_C > \rho_D$ then in a mutation-selection equilibrium, in the limit of weak selection and low mutation, there are more cooperators than defectors. The condition $\rho_C > \rho_D$ is given by

$$\prod_{i=1}^{N-1} \frac{g_i}{f_i} < 1 \tag{30}$$

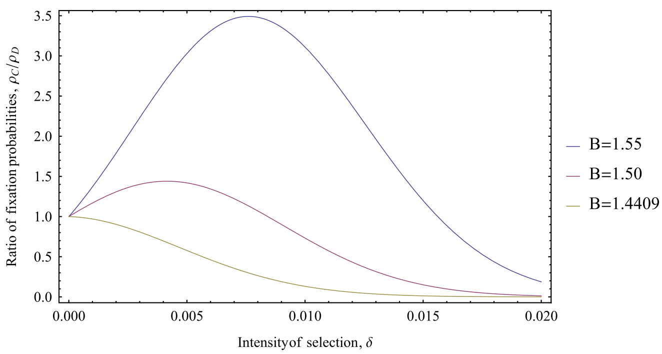

Fig. 2. The ratio of the fixation probabilities, ρ_C/ρ_D, is shown versus the intensity of selection, δ. For this choice of parameters we have $\rho_C=\rho_D$ in the limit of weak selection if $B=1.4409$. In this case the slope of the curve is zero at $\delta=0$. For $B=1.5$ and $B=1.55$ we have $\rho_C>\rho_D$ in the limit of weak selection and therefore the slope is positive at $\delta=0$. Other parameter values are as follows: population size $N=1000$, average number of rounds per individual $h=10$, probability of identification of defector, $Q=0.3$, cost of cooperation $c=1$. As the intensity of selection, δ, reaches $-1/F_1=[N-1-(2Q/N)+hQ]/[ch(N-1)]=0.1003$, the fixation probability of a single cooperator, ρ_C, tends to 0.

In the limit $N\to\infty$ and using x for the frequency of cooperators we can write

$$F(x)=ch\left(Bx-\frac{1-x}{hQx+1}\right)$$
$$G(x)=ch\frac{(B+1)x}{hQx+1} \tag{31}$$

The condition $\rho_C>\rho_D$ can be calculated as

$$\int_0^1 [F(x)-G(x)]\,dx>0 \tag{32}$$

We obtain

$$BH(H-2)+2(B-H)\log(1+H)>0 \tag{33}$$

Fig. 2 shows the ratio of fixation probabilities, ρ_C/ρ_D, as a function of the intensity of selection, δ. If (33) holds then the slope of ρ_C/ρ_D at $\delta=0$ is positive. In this case there is an intermediate intensity of selection which maximizes ρ_C/ρ_D. As δ increases the ratio converges to zero at the point where the fitness of a single cooperator, f_1, becomes 0.

3.3. Deterministic game dynamics for infinite population size

Now we investigate deterministic evolutionary dynamics for the limit of infinite population size, $N\to\infty$ (Maynard Smith, 1982; Hofbauer and Sigmund, 1988a, 1998b, 2003; Weibull, 1995; Samuelson, 1997; Cressman, 2003; Nowak and Sigmund, 2004; Mailath and Samuelson, 2006; Perc and Szolnoki, 2010). As before the fraction of cooperators are given by $x=i/N$. The fraction of defectors are given by $1-x$. We have the payoff functions:

$$F(x)=ch\left(Bx-\frac{1-x}{hQx+1}\right)$$
$$G(x)=ch\frac{(B+1)x}{hQx+1} \tag{34}$$

Frequency dependent selection dynamics can be written as

$$\dot{x}=x(1-x)[F(x)-G(x)] \tag{35}$$

Omitting a factor ch, which only rescales time, we obtain

$$\dot{x}=x(1-x)\frac{BHx^2-1}{Hx+1} \tag{36}$$

If $BH<1$ then defectors dominate cooperators. If $BH>1$ we have bistability. There is an unstable interior equilibrium at $x^*=1/\sqrt{BH}$. If $BH>4$ then the all-cooperate equilibrium has the bigger basin of attraction.

4. Hesitation to cooperate

It is possible that cooperators hesitate to interact, because they might be uncertain about the reputation of the other person. Here we assume that cooperators interact with a probability P, if they have no adverse information about the other player. As before, cooperators do not interact if the reputation of the other person indicates a defector.

In the Appendix, we show that the expected number of exploitations done by a single defector who was offered k games with cooperators is $(1-(1-PQ)^k)/Q$. Using this result, we now calculate the expected number of exploitations done by defectors when there are k CD games. Using the same method as before we have

$$\begin{aligned}
e_k &= \sum_{k_1+\cdots+k_{N-i}=k}\left(\binom{k}{k_1,k_2,\ldots,k_{N-i}}\Big/(N-i)^k\right)\sum_{j=1}^{N-i}\frac{1-(1-PQ)^{k_j}}{Q}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1+\cdots+k_{N-i}=k}\binom{k}{k_1,k_2,\ldots,k_{N-i}}(1-(1-PQ)^{k_1})\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(1-(1-PQ)^{k_1})\sum_{k_2+\cdots+k_{N-i}=k-k_1}\\
&\quad\times\binom{k-k_1}{k_2,k_3,\ldots,k_{N-i}}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(1-(1-PQ)^{k_1})(N-i-1)^{k-k_1}\\
&=\frac{N-i}{Q(N-i)^k}\sum_{k_1=0}^{k}\binom{k}{k_1}(N-i-1)^{k-k_1}-(1-PQ)^{k_1}(N-i-1)^{k-k_1}\\
&=\frac{N-i}{Q(N-i)^k}[(N-i)^k-(N-i-PQ)^k]\\
&=\frac{N-i}{Q}\left[1-\left(1-\frac{PQ}{N-i}\right)^k\right]
\end{aligned} \tag{37}$$

The expected payoff for each cooperator after m rounds is

$$\begin{aligned}
F_i(m) &= \frac{1}{i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^l(2j(b-c)P^2-ce_k)\\
&=\frac{1}{i}\left(2mP^2(b-c)p-\frac{c(N-i)}{Q}\sum_{j+k+l=m}\binom{m}{j,k,l}\right.\\
&\quad\left.\times p^jq^kr^l\left(1-\left(1-\frac{PQ}{N-i}\right)^k\right)\right)\\
&=\frac{1}{i}\left(2mP^2(b-c)p-\frac{c(N-i)}{Q}\left(1-\left(p+q\left(1-\frac{PQ}{N-i}\right)+r\right)^m\right)\right)\\
&=\frac{1}{i}\left(2mP^2(b-c)p-\frac{c(N-i)}{Q}\left(1-\left(1-\frac{qPQ}{N-i}\right)^m\right)\right)
\end{aligned} \tag{38}$$

Similarly, the expected payoff for each defector after m rounds is

$$\begin{aligned}
G_i(m) &= \frac{1}{N-i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^lbe_k\\
&=\frac{1}{N-i}\sum_{j+k+l=m}\binom{m}{j,k,l}p^jq^kr^lb\frac{N-i}{Q}\left(1-\left(1-\frac{PQ}{N-i}\right)^k\right)\\
&=\frac{b}{Q}\left(1-\left(1-\frac{qPQ}{N-i}\right)^m\right)
\end{aligned} \tag{39}$$

Note that in order to have exactly m rounds, the game has to continue for the first $(m-1)$ rounds and then stop at round m. This happens with probability $(1-w)w^{m-1}$. Therefore the expected payoff for each cooperator for the whole game is

$$F_i = \sum_{m=1}^{\infty}(1-w)w^{m-1}F_i(m)$$
$$= \frac{(1-w)}{i}\sum_{m=1}^{\infty} w^{m-1}\left(2mP^2(b-c)p - \frac{c(N-i)}{Q}\left(1-\left(1-\frac{qPQ}{N-i}\right)^m\right)\right)$$
$$= \frac{(1-w)}{i}\left(\frac{2(b-c)P^2p}{(1-w)^2} - \frac{c(N-i)}{Q}\left(\frac{1}{1-w} - \frac{1-\frac{qPQ}{N-i}}{1-\left(1-\frac{qPQ}{N-i}\right)w}\right)\right)$$
$$= \frac{(1-w)}{i}\left(\frac{2(b-c)P^2p}{(1-w)^2} - \frac{c(N-i)}{Q}\frac{\frac{qPQ}{N-i}}{(1-w)\left(1-\left(1-\frac{qPQ}{N-i}\right)w\right)}\right)$$
$$= \frac{1}{i}\left(\frac{2(b-c)P^2p}{1-w} - \frac{cPq}{1-\left(1-\frac{qPQ}{N-i}\right)w}\right). \quad (40)$$

Here we have used Eqs. (22) and (23).

Similarly, we can find the expected payoff for each defector for the whole game

$$G_i = \sum_{m=1}^{\infty}(1-w)w^{m-1}G_i(m)$$
$$= \frac{b(1-w)}{Q}\sum_{m=1}^{\infty} w^{m-1}\left(1-\left(1-\frac{qPQ}{N-i}\right)^m\right)$$
$$= \frac{b(1-w)}{Q}\left(\frac{1}{1-w} - \frac{1-\frac{qPQ}{N-i}}{1-\left(1-\frac{qPQ}{N-i}\right)w}\right)$$
$$= \frac{b(1-w)}{Q}\left(\frac{\frac{qPQ}{N-i}}{(1-w)\left(1-\left(1-\frac{qPQ}{N-i}\right)w\right)}\right)$$
$$= \frac{bPq}{(N-i)\left(1-\left(1-\frac{qPQ}{N-i}\right)w\right)} \quad (41)$$

For continuation probability w, the average number of rounds is given by $M = 1/(1-w)$. The average number of rounds offered to each player is $h = 2M/N$. Therefore we have $w = 1-2/(hN)$. Again we use $B = (b/c)-1$. Together with the relations $p = \binom{i}{2}/\binom{N}{2}$ and $q = i(N-i)/\binom{N}{2}$, we can express F_i and G_i as

$$F_i = chP\left(\frac{BP(i-1)}{N-1} - \frac{N-i}{N-1-2PQ\frac{i}{N}+hQi}\right)$$
$$G_i = chP\frac{(B+1)i}{N-1-2PQ\frac{i}{N}+hPQi} \quad (42)$$

Let us now discuss some properties of this payoff function. First we note that everyone being a defector is always a strict Nash equilibrium. The expected payoff for a single cooperator in a population of defectors is negative, $F_1 < 0$, while the payoff for defectors in an all-defector population is $G_0 = 0$.

Next we investigate the condition for cooperation to be a strict Nash equilibrium. If everyone is a cooperator, then the payoff for each individual is $F_N = chBP^2$. If one player switches to defection, his payoff becomes

$$G_{N-1} = \frac{chP(B+1)}{1-\frac{2PQ}{N}+hPQ} \quad (43)$$

From $F_N > G_{N-1}$ we derive

$$h > \frac{1}{BP^2Q} + \frac{1-P}{P^2Q} + \frac{2}{N} \quad (44)$$

For large N and using $H=hQ$ the condition becomes

$$B(HP^2+P-1) > 1 \quad (45)$$

Thus $HBP^2 > 1$ is a necessary condition, which can also be written as $P > 1/\sqrt{HB}$. The probability P that cooperators choose to interact, if they have no adverse information about the other player, has to exceed a minimum value for cooperation to be a strict Nash equilibrium.

5. Malicious gossip

We now consider the case where a cooperator is sometimes wrongly accused as a defector due to malicious gossip. Suppose this happens with a fixed probability v each round regardless of whether this individual was chosen for interaction. In a population of only cooperators, the expected number of people who receive a wrong reputation in the first round is $V=Nv$.

If everyone is a cooperator, then in the mth round, the game is accepted by both players with probability $(1-v)^{m-1}(1-v)^{m-1} = (1-v)^{2m-2}$. The probability to have the mth round is w^{m-1}. The probability that the mth round exists and the game is accepted is $(1-v)^{2m-2}w^{m-1}$. The expected payoff of a cooperator when everyone else is also a cooperator is

$$F_N = \frac{1}{N}\sum_{m=1}^{\infty} 2(b-c)(1-v)^{2(m-1)}w^{m-1}$$
$$= \frac{2(b-c)}{N}\frac{1}{1-(1-v)^2w}$$
$$= \frac{2(b-c)}{N}\frac{1}{1-(1-v)^2\left(1-\frac{2}{hN}\right)} \quad (46)$$

A defector accepts to play the game even if the reputation of the other person is that of a defector. Therefore, a defector can still exploit a cooperator who has a bad reputation because of malicious gossip. Thus, the expected payoff of a single defector when everyone else is a cooperator does not change by malicious gossip; it is the same as Eq. (25) and given by

$$G_{N-1} = \frac{c(B+1)h}{1+hQ-\frac{2Q}{N}} \quad (47)$$

Cooperation is a strict Nash equilibrium if $F_N > G_{N-1}$, which leads to

$$\frac{2B}{N}\frac{1}{1-(1-v)^2\left(1-\frac{2}{hN}\right)} > \frac{(B+1)h}{1+hQ-\frac{2Q}{N}} \quad (48)$$

This condition is equivalent to

$$h[2BQ - N(B+1)(2v-v^2)] > 2(B+1)(1-v)^2 - 2B + \frac{4BQ}{N} \quad (49)$$

Using $V=Nv$ we obtain

$$h\left[2BQ-(B+1)V\left(2-\frac{V}{N}\right)\right] > 2(B+1)\left(1-\frac{V}{N}\right)^2 - 2B + \frac{4BQ}{N} \quad (50)$$

For large N, we can write

$$h[BQ-(B+1)V] > 1 \quad (51)$$

In Fig. 3 we show the minimum number of rounds needed for cooperation to be a strict Nash equilibrium as a function of the probability of wrong accusation, v, per round per individual. For cooperation to be a strict Nash equilibrium, the probability, v,

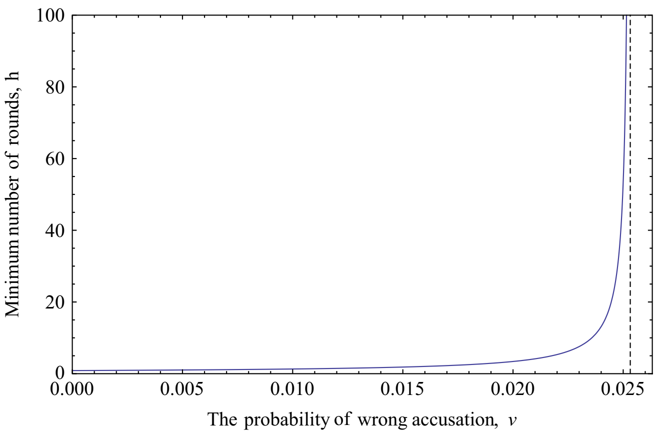

Fig. 3. The minimum number of rounds per individual, h, that is needed for cooperation to be a strict Nash equilibrium, is shown as a function of the probability of wrong accusation, v. The parameter values are as follows: population size $N=10$, probability of identification of a defector, $Q=0.3$ and benefit to cost ratio, $B=b/c-1=5$. Note that cooperation cannot be a strict Nash equilibrium if $v \geq 1-\sqrt{1-(2BQ/N(B+1))}$ (broken line).

must be below a critical value:

$$v < 1-\sqrt{1-\frac{2BhQ-2-\frac{4BQ}{N}}{(B+1)(hN-2)}} \tag{52}$$

Even if the number of rounds, h, goes to infinity we still need

$$v < 1-\sqrt{1-\frac{2BQ}{N(B+1)}} \tag{53}$$

There is an upper threshold for the amount of malicious gossip that is compatible with evolution of cooperation by indirect reciprocity.

6. Conclusion

We have proposed a new model of indirect reciprocity with optional interactions (or partner choice). In any one round an interaction between two individuals is offered. An interaction occurs if both players accept. Cooperators accept unless the reputation of the other player is that of a defector. If it happens that a cooperator interacts with a defector whose reputation is still unknown, then there is a certain probability that the reputation of the defector will be established after the interaction. Therefore, as the game progresses more and more defectors will become known and they lose the possibility to further exploit cooperators. We calculate the minimum number of games that is needed for cooperators to be favored over defectors. This critical threshold depends on the benefit-to-cost ratio, the average number of rounds per player, the population size and the probability that the identity of a defector is revealed after exploiting a cooperator. Our game has a nonlinear fitness function. We study Nash equilibrium arguments and evolutionary dynamics for both finite and infinite population size. We have also studied two extensions taking into account uncertainty about the reputation of the other player.

A simple summary of our findings is as follows. For large population size indirect reciprocity with optional interaction can promote cooperation if

$$hQB > 1 \tag{54}$$

Here $B=(b/c)-1$ characterizes the benefit-to-cost ratio, while h is the average number of interactions per individual and Q is the identification probability of defectors.

Appendix A

A.1. Possible rankings of the fixation probabilities

The fixation probability of a single cooperator is ρ_C and that of a single defector is ρ_D. In the limit of weak selection, $\delta \to 0$, and large population, $N \to \infty$, with $N\delta \to 0$, we have for our game the following implications:

$$(\rho_C > 1/N) \Rightarrow (\rho_C > \rho_D) \Rightarrow (\rho_D < 1/N) \tag{55}$$

Thus, the following rankings are possible:

$$\begin{aligned} &\rho_C > 1/N > \rho_D \\ &1/N > \rho_C > \rho_D \\ &1/N > \rho_D > \rho_C \\ &\rho_D > 1/N > \rho_C \end{aligned} \tag{56}$$

Impossible are rankings where both ρ_C and ρ_D are greater than $1/N$. For each finite population of size N, the possible rankings of fixation probabilities in the limit of weak selection are the same as for the infinite population. We present proofs for possible orders in following subsections.

A.1.1. Proof for infinite population size

The condition $\rho_C > \rho_D$ can be calculated as

$$\int_0^1 [F(x)-G(x)]\,dx > 0 \tag{57}$$

We obtain

$$BH(H-2)+2(B-H)\log(1+H) > 0 \tag{58}$$

This leads to

$$B > \frac{2H\log(1+H)}{H(H-2)+2\log(1+H)} \tag{59}$$

The condition $\rho_C > 1/N$ can be calculated as

$$\int_0^1 (1-x)[F(x)-G(x)]\,dx > 0 \tag{60}$$

We obtain

$$B > \frac{6H(1+H)\log(1+H)-6H^2}{6(1+H)\log(1+H)-H(6+3H-H^2)} \tag{61}$$

The condition $\rho_D < 1/N$ can be calculated as

$$\int_0^1 x[F(x)-G(x)]\,dx > 0 \tag{62}$$

We obtain

$$B > \frac{-6H\log(1+H)+6H^2}{6H-3H^2+2H^3-6\log(1+H)} \tag{63}$$

By direct computation, we note that

$$\begin{aligned} &\frac{6H(1+H)\log(1+H)-6H^2}{6(1+H)\log(1+H)-H(6+3H-H^2)} \\ &\quad > \frac{2H\log(1+H)}{-2H+H^2+2\log(1+H)} \\ &\quad > \frac{-6H\log(1+H)+6H^2}{6H-3H^2+2H^3-6\log(1+H)} \end{aligned} \tag{64}$$

Therefore, in the limit of weak selection and when population size tends to infinity, we have the following implication relations:

$$(\rho_C > 1/N) \Rightarrow (\rho_C > \rho_D) \Rightarrow (\rho_D < 1/N) \tag{65}$$

A.1.2. Proof for finite population size

From (28) and (29), we have a linear approximation of the fixation probabilities as follows:

$$\rho_C \approx \frac{1}{N} + \frac{\delta}{N^2}((N-1)(F_1-G_1)+(N-2)(F_2-G_2)+\cdots+(F_{N-1}-G_{N-1}))$$
$$\rho_D \approx \frac{1}{N} - \frac{\delta}{N^2}((F_1-G_1)+2(F_2-G_2)+\cdots+(N-1)(F_{N-1}-G_{N-1})) \quad (66)$$

The difference is given by

$$\rho_C - \rho_D \approx \frac{\delta}{N}((F_1-G_1)+(F_2-G_2)+\cdots+(F_{N-1}-G_{N-1})) \quad (67)$$

In the limit of weak selection, $\rho_C > 1/N$ holds iff

$$(N-1)(F_1-G_1)+(N-2)(F_2-G_2)+\cdots+(F_{N-1}-G_{N-1}) > 0. \quad (68)$$

Furthermore, $\rho_C > \rho_D$ holds iff

$$(F_1-G_1)+(F_2-G_2)+\cdots+(F_{N-1}-G_{N-1}) > 0. \quad (69)$$

Furthermore, $\rho_D < 1/N$ holds iff

$$(F_1-G_1)+2(F_2-G_2)+\cdots+(N-1)(F_{N-1}-G_{N-1}) > 0. \quad (70)$$

From (42), we have

$$F_i - G_i = ch\left(\frac{B(i-1)}{N-1} - \frac{N+Bi}{N-1-2Q\frac{i}{N}+hQi}\right). \quad (71)$$

We can show that $F_i - G_i$ increases as i increases from 1 to $N-1$. Therefore, by Chebyshev's sum inequality, we have

$$\sum_{i=1}^{N-1} i(F_i-G_i) \geq \frac{N}{2}\sum_{i=1}^{N-1}(F_i-G_i) \geq \sum_{i=1}^{N-1}(N-i)(F_i-G_i). \quad (72)$$

From the above inequality, we directly obtain

$$(\rho_C > 1/N) \Rightarrow (\rho_C > \rho_D) \Rightarrow (\rho_D < 1/N) \quad (73)$$

in the limit of weak selection ($\delta \to 0$) for any population size, N, as desired.

A.2. Expected number of exploitations done by a single defector

For the hesitation model, we show that the expected number of exploitations done by a single defector who was offered k games with cooperators is $[1-(1-PQ)^k]/Q$. We will use mathematical induction on k.

For $k=1$ the defector exploits a cooperator if the cooperator accepts the game. Since the cooperator does not know the identity of the defector, the only barrier of this interaction is the hesitation of the cooperator. The cooperator accepts the game with probability P. Thus exploitation occurs with probability P. Therefore the expected number of exploitations done by a single defector who was offered a single game is P, as desired.

Let us now assume that the formula is correct for k and let us prove it for $k+1$. These are the following cases according to the outcome of the first round:

(i) The defector manages to exploit a cooperator in his first game and the identity of defector is revealed. This happens with the probability PQ. In this case no further exploitation is possible. Hence the contribution to the expected number of exploitations from this case is

$$PQ \quad (74)$$

(ii) The defector manages to exploit a cooperator in his first game, and the identity of the defector is not revealed. This happens with probability $P(1-Q)$. After his first game, the defector is going to continue the game as if he did not have any previous game. Therefore the situation from the second round on is exactly the same as if he was offered k games in the beginning. Therefore the contribution to the expected number of exploitations from this case is

$$P(1-Q)\left(1+\frac{1-(1-PQ)^k}{Q}\right) \quad (75)$$

(iii) The defector could not exploit a cooperator in his first game, because the cooperator rejected to play due to hesitation. This happens with the probability $1-P$. After his first game, the defector is in exactly the same situation as if he was offered k games in the beginning. Therefore the contribution to the expected number of exploitations from this case is

$$(1-P)\frac{1-(1-PQ)^k}{Q} \quad (76)$$

In total we have

$$PQ + P(1-Q)\left(1+\frac{1-(1-PQ)^k}{Q}\right) + (1-P)\frac{1-(1-PQ)^k}{Q} = \frac{1-(1-PQ)^{k+1}}{Q} \quad (77)$$

as desired. This completes the mathematical induction.

A.3. Discussion of D–D interactions

We can also think of a model where a defector's identity is revealed with a certain probability when a defector plays against another defector. In this model it is hard to calculate the payoffs of defectors, because we need to keep track not only of the number of D–D interactions, but also of the specific defectors in D–D interactions and the order of those interactions. For example, the game sequence $(D_1 - C_1)$ followed by $(D_1 - D_2)$ and the game sequence $(D_1 - D_2)$ followed by $(D_1 - C_1)$ lead to different probabilities for the exploitation of the cooperator. In the first sequence, D_1 exploits C_1 with probability one and in the second sequence, D_1 exploits C_1 only when his identity is not revealed after the $(D_1 - D_2)$ round. Note that this modification will not change the conditions for Nash equilibrium, where we only have a single defector in the population. We leave the general investigation of this case for future research.

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